Introducing risk into a Tobin asset-allocation model

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The Tobin Model

- Linearized and normalised by total wealth:

$$\begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix} = \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \vdots \\ \lambda_{n0} \end{pmatrix} + \begin{pmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nn} \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix},$$

where: s_i are the share of total wealth allocated to asset *i*, r_i are the return rate, λ_{i0} is "base share"; and the λ_{ij} are marginal substitution coefficient related to returns change

Tobin Model, part II

- Base share satisfy: $\sum_{j=1}^{n} \lambda_{ij} = 0.$
- The λ_{ij} satisfy: $\sum_{i=1}^{n} \lambda_{ij} = 0,\ j > 0.$
- these criteria ensure that both the observed shares s_i and the base shares do indeed act like shares, by adding up to one
- If observed shares equal the base shares when all assets have the same return imply: $\sum_{i=1}^{n} \lambda_{i0} = 1,$
- Symmetry condition: $\lambda_{ij} = \lambda_{ji}.$

Some more notation

- Tobin's q: $q_i = \dfrac{s_i W}{K_i}$,
- Average *q* applied to the whole economy:

$$\bar{q} = (1-m)\frac{W}{K}, \quad K = \sum_{i=1}^{n-1} K_i .$$

- Re-writing and solving for assets share:

$$s_i = (1-m)\frac{K_i}{K}\frac{q_i}{\bar{q}}, \quad i = 1, 2, \ldots, n-1, \qquad s_n = m.$$

- If q_i is equal to one, then w_i=s_i:

$$w_i = (1-m^*)\frac{K_i}{K}, \quad i = 1, 2, \ldots, n-1, \qquad w_n = m^*.$$

Indistinguishable assets

- Start with only different return
  - All wealth into highest return bearing asset
  - Market price rises and return decreases
- At equilibrium: returns are equalised and Tobin's *q* are equal to 1, we thus have: $\lambda_{i0} = w_i$.
- What if returns on asset *j* increase by Δr_j?
  - Cannot shift all wealth into that asset
  - Hence importance of absorptive capacity of asset *j.*
  - We propose: $\lambda_{ij} = a\left(\delta_{ij} w_i - w_i w_j\right)$, where a measures how much investors weight high returns over turnover

Introducing Risk

- tendency for returns to differ from an expectation, whether of a historical mean for a particular asset or the return for a benchmark asset
  - individual asset – absolute measure
  - or compared to the benchmark – relative measure

Risk indicators

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- Large number of risk measures available (Schulmerich, Leporcher, and Eu 2015)
- Bunn and Campbell (2015), professional investment managers, strongly recommend against the standard academic metrics
- We follow conventional academic economics, and dominant current practice, by assuming that investors assign each asset *i* a risk value v_i independent of other assets.
- We also assume no relationship between risk and return – the correlation between them can be positive, negative, or zero.

The Model

- Characterize the market by two parameters:
  - tolerance for risk, v^*,
  - preference for keeping turnover low, φ, as against maximizing returns
- Model made operational by
  - maximizing a weighted sum of average returns and the deviation from the equilibrium shares w_i
  - subject to a constraint that enforces risk tolerance

Deviation from weights

- Investors will allocate wealth in proportion to the distribution of the book value of underlying assets, deviating only to the extent that risk and return deviates between assets

- Deviation function: $D = \sum_{i=1}^{n} w_i f\left(\frac{s_i}{w_i} - 1\right).$

- Investors apply a criterion of minimizing deviation between market capitalization and book value independent of the underlying asset

Model, formalised

- Taking the Taylor expansion from the deviation function, we get: $D \cong r^* \sum_{i=1}^{n} w_i \left(\frac{s_i}{w_i} - 1 \right)^2.$
- Objective function: $-\varphi r^* \sum_{i=1}^{n} w_i \left(\frac{s_i}{w_i} - 1 \right)^2 + (1-\varphi) \sum_{i=1}^{n} s_i r_i \cdot$
- Risk tolerance: $\sum_{i=1}^{n} s_i v_i = v^*.$
- Shares are share: $\sum_{i=1}^{n} s_i = 1, \quad s_i \geq 0.$
- Lagrangian:

$$Z = -\varphi r^* \sum_{i=1}^{n} w_i \left(\frac{s_i}{w_i} - 1 \right)^2 + (1-\varphi) \sum_{i=1}^{n} s_i r_i + \lambda \left(1 - \sum_{i=1}^{n} s_i \right) + \mu \left(v^* - \sum_{i=1}^{n} s_i v_i \right) + \sum_{i=1}^{n} \theta_i \left(a_i^2 - s_i \right).$$

Solution

- First order conditions:

$$\frac{\partial Z}{\partial s_i} = -2r^*\varphi(s_i - w_i) + (1-\varphi)w_i r_i - \lambda w_i - \mu w_i v_i - \theta_i w_i = 0, \quad \frac{\partial Z}{\partial a_i} = 2\theta_i a_i = 0.$$

- Expression for the shares:

$$s_i = w_i\left(1 + \frac{1-\varphi}{2\varphi}\frac{r_i}{r^*} - \frac{\lambda + \mu v_i}{2r^*\varphi}\right).$$

- Using that shares must sum to one and that the portfolio is built with a specified average risk, and solving we get…

Solution for the shares

$$s_i = w_i \left[1 + \frac{\left(v_i - \langle v \rangle\right)\left(v^* - \langle v \rangle\right)}{\langle v^2 \rangle - \langle v \rangle^2} \right] + w_i a \left[r_i + \sum_{j=1}^{n} w_j r_j \left(\frac{\langle v \rangle v_j - v_i v_j}{\langle v^2 \rangle - \langle v \rangle^2} - \frac{\langle v^2 \rangle - v_i \langle v \rangle}{\langle v^2 \rangle - \langle v \rangle^2} \right) \right],$$

where $\langle r \rangle = \sum_{i=1}^{n} w_i r_i, \quad \langle rv \rangle = \sum_{i=1}^{n} w_i r_i v_i, \quad \langle v \rangle = \sum_{i=1}^{n} w_i v_i, \quad \langle v^2 \rangle = \sum_{i=1}^{n} w_i v_i^2,$

and $a = \dfrac{1-\varphi}{2r^* \varphi}.$

Recovering the Tobin model

- Base shares: $\lambda_{i0} = w_i + w_i \frac{\left(v_i - \langle v \rangle\right)\left(v^* - \langle v \rangle\right)}{\langle v^2 \rangle - \langle v \rangle^2}.$
- Substitution coefficients:

$$\lambda_{ij} = a\left[\delta_{ij} w_i - w_i w_j \frac{\langle v^2 \rangle - \langle v \rangle \left(v_i + v_j\right) + v_i v_j}{\langle v^2 \rangle - \langle v \rangle^2}\right].$$

- They respect the various constraints
- In the limit that all risk measures converge to a common value *v*: $\lim_{v_i \to v} \lambda_{i0} \to w_i,$

$$\lim_{v_i \to v} \lambda_{ij} \to a\left(\delta_{ij} w_i - w_i w_j\right).$$

Applications

- Finding an expression for Tobin's *q*
- Macro-prudential policies and capital adequacy ratio
- The impact of quantitative easing and green QE
- Empirics

Tobin's q

- Rearranging the solution of the optimisation problem for the shares:

$$\frac{s_i}{w_i} = 1 + a\frac{\left(\langle v^2\rangle - \langle v\rangle^2\right)\left(r_i - \langle r\rangle\right) + \left(v_i - \langle v\rangle\right)\left(\langle v\rangle\langle r\rangle - \langle rv\rangle\right)}{\langle v^2\rangle - \langle v\rangle^2} + \frac{\left(v_i - \langle v\rangle\right)\left(v^* - \langle v\rangle\right)}{\langle v^2\rangle - \langle v\rangle^2}.$$

- Using $\sigma_x^2 = \langle x^2\rangle - \langle x\rangle^2, \quad \rho_{rv} = \dfrac{\langle rv\rangle - \langle v\rangle\langle r\rangle}{\sigma_r\sigma_v},$
- Defining $z_i \equiv \dfrac{v_i - \langle v\rangle}{\sigma_v}.$ as a risk deviation index
- And $R \equiv \dfrac{v^* - \langle v\rangle}{\sigma_v},$ as an indicator for the state of the market

Interpretation

- Final formulation

$$\frac{1-m}{1-m^*}\frac{q_i}{\bar{q}} = 1 + a\left(r_i - \langle r\rangle\right) + \left(R - a\frac{\sigma_r}{\sigma_v}\rho_{rv}\right) z_i,$$

$$\frac{m}{m^*} = 1 + a\left(r_n - \langle r\rangle\right) + \left(R - a\frac{\sigma_r}{\sigma_v}\rho_{rv}\right) z_n.$$

- In recession, R is negative: flight to liquidity
- Changes in targeted risk, impact R and reallocation between *q*
- Correlation between return and risk and leads to re-allocation between assets depending on the sign

Macro-prudential policy

- Impacts of Capital Adequacy Ratios on the economy (BCBS, 2016)
  - Reduced lending directly
  - Reduced lending via increase in lending rate

	Lending reduction (%)	Credit growth reduction (%)	Sample	Estimation period	Period of the accumulated effect (months)
MAG (2010)	1.4		Average 15 countries		24
Fraisse et al (2015)	1–8		France	2008–2011	12
Aiyar et al (2014b)		4.6	UK	1998–2007	<3
Bridges et al (2014)	3.5		UK	1990–2011	36
Messonier and Monks (2014)		1.2	France	2011–2012	9
Noss and Toffano (2014)	1.4		UK	1986–2010	Long run
Meeks (2014)	0.2 (mortgage) 0.5 (corporate)		UK	1989–2008	Long run
Mendicino et al (2015)2	0.15 (mortgage) 0.43 (corporate)		Euro area	2001–2013	Long run
Sutorova and Teply (2013)	1.4–3.5	1.2–4.6	Europe	2006–2011	Long run
De-Ramon et al (2012)	1.6		UK	1992–2010	Long run

[1] 1% at the intensive margin, 8% considering both the intensive and extensive margins. [2] Authors' calculations.

Interest rate lending channel

- Under Modigliani-Miller, there should be no impact of a rise in CAR on the funding costs
- But many reasons why M-M would not work (Elliott 2013, Miles et al. 2013, Allen and Carletti 2013)
- Anyhow, even if no change on funding costs, increases in CAR are likely to impact return on equity (RoE) since it is unlikely to impact profits proportionally to equity, hence need to rise profits by increasing the lending rate.

Our contribution to the debate

- What happens when using out risk-weighted Tobin model?
- If banks look at their *q* and if we assume that a change in CAR impacts the risk perception of the bank's traded equity, assuming small bank with respect to the market

$$\frac{\partial\, q_i}{\partial\, CAR_i} = a\left((1-w_i)\frac{\partial\, r_i}{\partial\, CAR_i} - r_i\frac{\partial\, w_i}{\partial\, CAR_i}\right) + \frac{\partial\, z_i}{\partial\, CAR_i}\left(R - a\frac{\sigma_r}{\sigma_v}\rho_{rv}\right)$$

Change in dividends to distribute?

- $\frac{\partial z_i}{\partial CAR_i} = \frac{\partial v_i}{\partial CAR_i} < 0,$
- Sign of R depends on the characteristic of the market: if bull R>>0, if bear R<<0,
- $\frac{\partial w_i}{\partial CAR_i} > 0,$
- Assuming ρ_{rv}=0 (i.e. no correlation between return and risk)

$$\frac{\partial q_i}{\partial CAR_i} = a\left((1 - w_i)\frac{\partial r_i}{\partial CAR_i} - r_i\frac{\partial w_i}{\partial CAR_i}\right) + \frac{\partial z_i}{\partial CAR_i}R$$

Banks' response to CAR?

- If banks look at their *q*
  - in a booming phase, banks will have to increase return, even when accounting for a decrease in risk (return trumps risk)
  - in a recession case, it is not necessarily the case, banks might actually be able to decrease their return (risk trumps return)

Conclusion

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- Practical matter: our model considerably simplifies the use of the Tobin model in simulation models.
- Theoretical level, the specification explicitly incorporates risk into the Tobin model, thereby reflecting the common understanding that investors look mainly at the risk-return profile of assets.
- Highlighted the importance of flight to liquidity in the specific case of Capital Adequacy Ratio

Next steps

- Further the CAR analysis
- QE and Green QE
- Empirics

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Thank you!

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